STD XII COMPETENCY BASED QUESTIONS

NAME OF TOPIC: THREE-DIMENSIONAL GEOMETRY

EXPECTED LEARNING OUTCOMES:

i) To find direction cosines and direction ratios of a line
ii) To find the equation of a line in space
iii) To find the angle between two lines
iv) To find the shortest distance between two lines

CORE CONCEPTS AND MAJOR AREAS:

Direction cosines and direction ratios of a line joining two points. Cartesian equation and vector equation of a line, skew lines, shortest distance between two lines. Angle between two lines.

QUESTIONS

MCQ

1 If a line makes angles α, β and γ with x, y and z-axes respectively, then the value of cos 2α + cos 2β + cos 2γ + 1 is

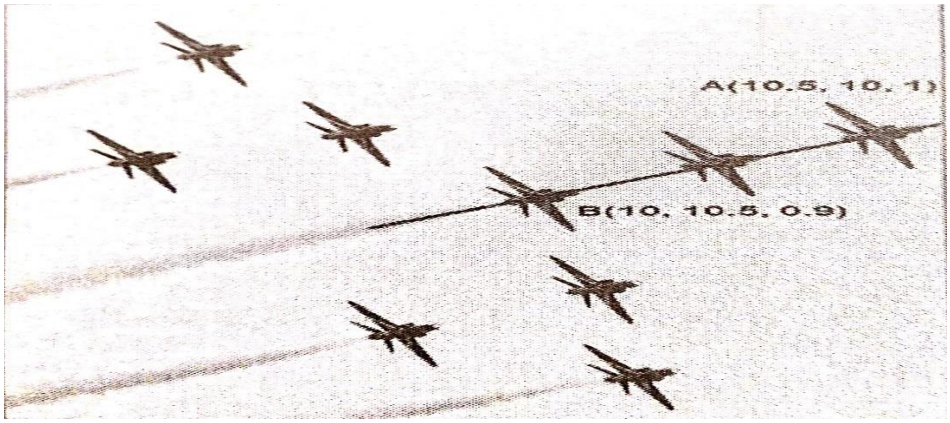
a) 1 b) -1 c) 2 d) 0

2 The equation of the line in vector form passing through the points (-1, 3, 5) and parallel to the line $\frac{x-2}{3} = \frac{y+1}{3} = \frac{z-3}{-2}$ is

a) $\vec{r} = \hat{\imath} - 3\hat{\jmath} - 5\hat{k} + \lambda (3\,\hat{\imath} + 3\,\hat{\jmath} - 2\,\hat{k})$
b) $\vec{r} = -\hat{\imath} + 3\hat{\jmath} + 5\hat{k} + \lambda (3\,\hat{\imath} + 3\,\hat{\jmath} - 2\,\hat{k})$
c) $\vec{r} = 3\hat{\imath} + 3\,\hat{\jmath} - 2\hat{k} + \lambda (-2\hat{\imath} + \hat{\jmath} - 3\,\hat{k})$
d) $\vec{r} = 3\hat{\imath} + 3\hat{\jmath} - 3\hat{k} + \lambda (2\,\hat{\imath} - 3\,\hat{\jmath} + 3\,\hat{k})$

CASE STUDY BASED QUESTION

1 Fighter jets are flying in a formation for an aero show as shown in the figure. Taking their control tower as the reference point and reference point being origin, the coordinates of two fighters in flight path are A (10.5 km, 10 km, 1 km) and B (10 km, 10.5 km, 0.9 km). They are moving along the straight line joining A and B at that point as seen in the figure



Based on the above information, answer the following questions.

i) What are the direction ratios and direction cosines of the line $\overleftrightarrow{AB}$?

ii) What is the angle made by the line $\overleftrightarrow{AB}$ with the positive direction of z-axis?

iii) What is the Cartesian equation of the line passing through A and B?

OR

What is the vector equation of the line passing through A and B?

ASSERTION REASON BASED QUESTIONS

In the following questions, a statement of Assertion(A) is followed by a statement of Reason (R).

Choose the correct answer out of the following choices

a. Both A and R are true and R is the correct explanation of A.

b. Both A and R are true and R is not the correct explanation of A.

c. A is true but R is false.

d. A is false but R is true.

1 **Assertion (A)**: If the points (3,2,2), (2,3,4) and (1, λ-2,6) and (3,1,5) are collinear, then λ=6

Reason (R): Three points A, B and C are collinear if direction ratios of AB and BC are proportional.

2 **Assertion (A)**: Lines $\frac{3-x}{2} = \frac{2y+4}{\lambda} = \frac{z-1}{5}$ and $\frac{x-2}{-1} = \frac{y+2}{4} = \frac{z-2}{2}$ are perpendicular, if λ= - 4.

Reason (R): Two lines with direction ratios (a_1, b_1, c_1) and (a_2, b_2, c_2) are perpendicular if $a_1a_2+b_1b_2+c_1c_2=0$

ANSWERS

MCQ

1 d) 0

2 b) $\vec{r} = -1\hat{\imath} + 3\hat{\jmath} + 5\hat{k} + \lambda(3\hat{\imath} + 3\hat{\jmath} - 2\hat{k})$

CASE STUDY BASED QUESTION

1 i) The given points are A (10.5, 10, 1) and B (10, 10.5, 0.9).
Direction ratios of the line joining (x_1, y_1, z_1) and (x_2, y_2, z_2) is (a, b, c) = (x_2-x_1, y_2-y_1, z_2-z_1).
Hence direction ratios of $\overleftrightarrow{AB}$ are (-0.5, 0.5, -0.1)
Now direction cosines of $\overleftrightarrow{AB}$ are $\left(\frac{-0.5}{\sqrt{0.51}}, \frac{0.5}{\sqrt{0.51}}, \frac{-0.1}{\sqrt{0.51}}\right)$

1 ii) The direction cosines of $\overleftrightarrow{AB}$ are $\left(\frac{-0.5}{\sqrt{0.51}}, \frac{0.5}{\sqrt{0.51}}, \frac{-0.1}{\sqrt{0.51}}\right)$

$\Longrightarrow (\cos\alpha, cos\beta, cos\gamma) = (\frac{-0.5}{\sqrt{0.51}}, \frac{0.5}{\sqrt{0.51}}, \frac{-0.1}{\sqrt{0.51}})$

where α, β, γ are the angles made by the line $\overleftrightarrow{AB}$ with positive direction of x-axis, y-axis and z-axis.

So, Cos $\gamma = \frac{-0.1}{\sqrt{0.51}}$

$\Longrightarrow \gamma = \cos^{-1}(\frac{-0.1}{\sqrt{0.51}})$

1 iii) The Cartesian equation of a line joining (x_1, y_1, z_1) and (x_2, y_2, z_2) is $(\frac{x-x1}{x2-x1} = \frac{y-y1}{y2-y1} = \frac{z-z1}{z2-z1})$.

Hence the Cartesian equation of a line $\overleftrightarrow{AB}$ is $(\frac{x-10.5}{-0.5} = \frac{y-10}{0.5} = \frac{z-1}{-0.1})$

OR

The vector equation of the line joining the points whose position vectors are $\vec{a}$ and $\overrightarrow{b}$ *is*

$\vec{r} = \vec{a} + \lambda(\vec{b} - \overrightarrow{a})$

Here $\vec{a} = 10.5\hat{\imath} + 10\hat{\jmath} + \hat{k}$ and $\vec{b} = 10\hat{\imath} + 10.5\hat{\jmath} + 0.9\hat{k}$

Hence the vector equation of the line is

$\vec{r} = (10.5\hat{\imath} + 10\hat{\jmath} + \hat{k}) + \lambda(-0.5\hat{\imath} + 0.5\hat{\jmath} - 0.1\hat{k})$

ASSERTION REASON BASED QUESTIONS

1 a) Both A and R are true and R is the correct explanation of A.

2 d) A is false but R is true

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