Three-dimensional geometry

Expected learning outcomes.

- find the direction ratios and direction cosines of a given vector.

- find the equation of line passing through a point and parallel to a given vector.

- find the equation of a line passing through two given points.

- calculate the angle between two lines.

MCQ

1. If the direction cosines of a given line are $\frac{1}{k}, \frac{1}{k}, \frac{1}{k}$ then the value of k is

   a) $\frac{1}{\sqrt{2}}$ b) $\pm \frac{1}{\sqrt{3}}$ c)1   d) $\pm \sqrt{3}$

2. The angle between the lines $2x = 3y = -z$ and $6x = -y = -4z$

   a) 0^0 b) 30^0 c) 45^0 d) 90^0

Assertion Reason questions

In the following questions consist of two statements – Assertion(A) and Reason(R). Answer these questions by selecting the appropriate option given below:

   (a) Assertion is true, reason is true, reason is a correct explanation for assertion.
   (b) Assertion is true, reason is true, reason is not a correct explanation for assertion.
   (c) Assertion is true, reason is false.
   (d) Assertion is false, reason is true.

1. Assertion(A): Equation of a line passing through the points (1,2,3) and (3, -1,3) is
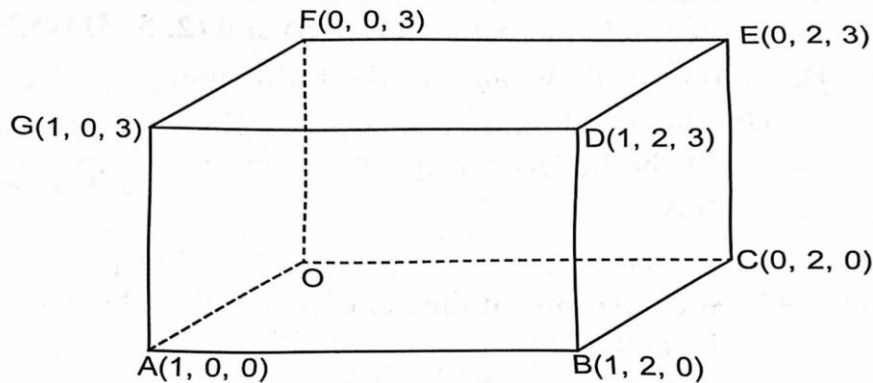 $\frac{x-3}{2} = \frac{y+1}{3} = \frac{z-3}{0}$

   Reason(R): Equation of a line passing through the points (x_1, y_1, z_1) and

   (x_2, y_2, z_2) is $\frac{x-x_1}{x_2-x_1} = \frac{y-y_1}{y_2-y_1} = \frac{z-z_1}{z_2-z_1}$.

2. Assertion(A): Direction cosines of y- axis are 0,1,0.
   Reason(R): Any order triplet represents direction cosines of a line.

Case Based Questions

Deepak made a cuboidal fish tank having coordinates O (0,0,0), A (1,0,0), B (1,2,0), C(0, 2,0), D(1,2,3), E(0,2,3), F(0,0,3) and G(1,0,3)

Based on the above information, answer the following:

    I.   Find the direction cosines of AB.
    II.   Write the cartesian equation of the diagonal OD.
    III.   Find the direction ratios of AB and BC.

Answers

MCQ

1. d) $\pm \sqrt{3}$
2. d) 90^0

Assertion Reason questions

1. d)
2. c)

Case Based Questions

    I.   Direction cosines of AB $= \dfrac{1-1}{\sqrt{0^2+2^2+0^2}}, \dfrac{2-0}{\sqrt{0^2+2^2+0^2}}, \dfrac{0-0}{\sqrt{0^2+2^2+0^2}}$

$$= \dfrac{0}{\sqrt{4}}, \dfrac{2}{\sqrt{4}}, \dfrac{0}{\sqrt{4}}$$

$$= 0, 1, 0$$

    II.   Write the cartesian equation of the diagonal OD.

$$\dfrac{x-x_1}{x_2-x_1} = \dfrac{y-y_1}{y_2-y_1} = \dfrac{z-z_1}{z_2-z_1}$$

$$\dfrac{x-0}{1-0} = \dfrac{y-0}{2-0} = \dfrac{z-0}{3-0}$$

$$\dfrac{x}{1} = \dfrac{y}{2} = \dfrac{z}{3}$$

III.   Direction ratios of AB are 1-1, 2 – 0, 0 – 0
                        0, 2, 0
        Direction ratios of BC are 0 -1, 2 -2, 0 – 0
                        -1, 0, 0