COMPETENCY BASED QUESTIONS

Polynomials

Learning outcomes:

- Understand the relationship between factors and roots of a polynomial.
- Find the roots (zeros) of a polynomial equation.
- Use factorization to solve polynomial equations.
- Apply polynomial concepts to solve problems in algebra, geometry, and other areas.
- Translate real-world problems into polynomial expressions or equations.

ASSERTION REASON

Directions: In the following questions a statement of **Assertion** is followed by a statement of **Reason**. Choose the correct option:

a) Both assertion and reason are true and reason is the correct explanation of assertion

b) Both assertion and reason are true but reason is not the correct explanation of assertion

c) Assertion is true but reason is false.

d) Assertion is false and reason is true.

Q1. Assertion: If the product of the zeroes of the quadratic polynomial

$p(x) = x^2 + 3x + 5k$ is -10 then value of k is -2.

Reason: Product of zeroes of a quadratic polynomial $p(x) = ax^2 + bx + c$ is c/a

Q2. Assertion: If α and β are the zeroes of the quadratic polynomial $p(x) = x^2 – ax - b$, then the value of $\alpha^2 + \beta^2$ is $a^2 + 2b$

Reason: If α and β are the zeroes of the above quadratic polynomial then $\alpha + \beta = a$ and $\alpha\beta = -b$ and $\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$

Q3. **Assertion:** If one zero of polynomial $p(x) = (k^2 + 4)x^2 + 13x + 4k$ is reciprocal of the other, then k=2.
Reason: If (x-a) is a factor of p(x), then p(a) = 0 i.e., a is a zero of p(x).

Q4. Assertion: If the sum of the zeroes of the quadratic polynomial $x^2-2kx+8$ is 2 then value of k is 1.
Reason: Sum of zeroes of a quadratic polynomial ax^2+bx+c is -b/a

Q5. Assertion: $3x^2$ + √3x + 2 is a quadratic polynomial with no real roots
Reason: b^2 - 4ac < 0 ⇒ Polynomial has no real roots.

Q6. Assertion: Sum and product of zeroes of a quadratic polynomial are √2 and $\sqrt{\frac{3}{2}}$ .Then the polynomial is $2x^2$ - √2x - $\sqrt{\frac{3}{2}}$
Reason: If α and β are the roots of a polynomial then x^2 - $(\alpha + \beta)x + \alpha\beta$ is the required polynomial.

Q7. Assertion: If one zero of polynomial p(x) = $(k^2 + 4)\ x^2$ + 13x + 4k is reciprocal of the other, then k = 2.
Reason: If (x - a) is a factor of p(x), then p(a) = 0 i.e., a is a zero of p(x).

Q8. Assertion: The graph y = f(x) is shown in figure, for the polynomial f(x). The number of zeroes of f(x) is 3.
Reason: The number of zeroes of the polynomial f(x) is the number of points of which f(x) cuts or touches the axes.

Q9. Assertion: A quadratic polynomial sum of whose zeroes is 8 and their product is 12 isx^2 – 20x + 96.
Reason: If α and β are the zeroes of the polynomial f(x), then the polynomial is given by f(x) = x^2 - (α + β) x + $\alpha\beta$.

MULTIPLE CHOICE QUESTIONS

Q10. If α, β are zeroes of x^2 – 3x + k, what is the value of k if 3α + 2β = 10?

a) -4 b) 4 c) -12 d) 12

Q11. If a polynomial p(x) = **ax^2 + bx +c has a + c = b then find one zero.**

a) $\frac{-b}{a}$ b) $\frac{b}{a}$ c) $\frac{c}{a}$ d) $\frac{-c}{a}$

Q12. Find the value of k if one zero of polynomial $3x^2$ = 8x + 2k is seven times the other.

a) 7/6 b) -7/6 c) 1/6 d) -1/6

Q13. The graph of the polynomial p(x) cuts the x axis at 3 points and touches it at 2 other points. The number of zeroes of p(x) is …

a) 1 b) 3 c) 4 d) 5

Q14. The graph of the polynomial p(x) intersects the x-axis three times in distinct points, and then which of the following could be an expression for p(x)?

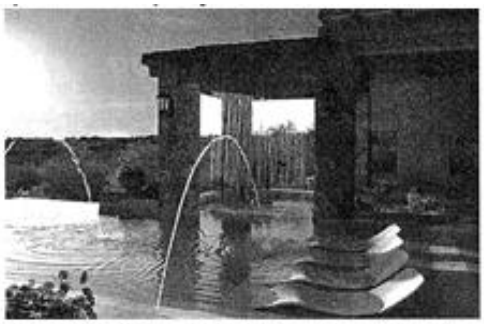
a) $4 -4x –x^2 +x^3$ b) $3x^2 +3x -3$ c) $3x+3$ d) $x^2 -9$

Q15. If each zero of the polynomial $x^2 +bx + c$ is decreased by 2, the resulting polynomial is $x^2– 2x + 1$, then

a) $b =6, c = 9$ b) $b =3, c= 5$ c) $b=2, c = -1$ d) $b =-4, c = 3$

CASE STUDY

Q16. Savitha everyday goes for swimming. One day she noticed water coming out of the pipes to fill the pool. She told her brother that shape of path of water falling is like a parabola.



Based on the given information, answer the following questions.

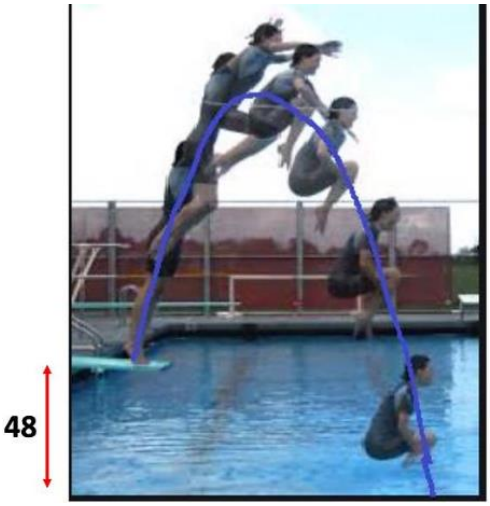
a) Determine the number of zeroes that polynomial $p(x) = (x - 2)^2 - 1$ can have.

b) Find the zeroes of the polynomial $p(x) = (x- 2)^2 -1$.

c) If α and β are the zeroes of the polynomial $p(x) = x^2 – 9$ then find the value of

a) $\alpha^2 + \beta^2$ b) $\frac{1}{\alpha} + \frac{1}{\beta}$

Q17. The figure shows the path of a diver Nitha when she takes a jump from the diving board. Nitha was standing on a diving board which is 48 feet above the water level. She took a dive into the pool. Her height (in feet) above the water level at any time ‘t’ in seconds is given by the polynomial $h(t) = -16t^2 + 8t + k$.

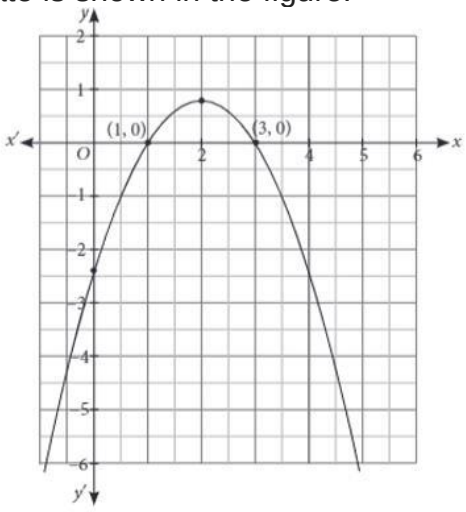


i) What is the value of k?
a) 0 b) -48 c) 48 d) 48/16

ii) At what time will Nitha touch the water in the pool?
a) 30seconds b) 2 seconds c) 1.5 seconds d) 0.5 seconds

iii) Another diver Rita's height (in feet) above the water level is given by another polynomial p(t) with zeroes −1 and 2. Then p(t) is given by
(a) $t^2 + t - 2$. (b) $t^2 + 2t - 1$ (c) $24t^2 - 24t + 48$. (d) $-24t^2 + 24t + 48$.

Q18. Quadratic polynomial can be used to model the shape of many architectural structures in the world. Priya visited a temple in Gwalior. On the way she sees the Agra Fort. The entrance gate of the fort has a shape of quadratic polynomial (parabolic). The mathematical representation of the gate is shown in the figure.



Based on the above information, answer the following questions.
(i) Find the zeroes of the polynomial represented by the graph.
(ii) What will be the expression for the polynomial represented by the graph?

(iii) What will be the value of the polynomial, represented by the graph, when x = 4?
(iv) If one zero of a polynomial p(x) = 7 and product of its zeroes is (-35) then find the expression for the polynomial p(x).

Q19. Naveeka everyday goes to swimming. One day, Naveeka noticed the water coming out of the pipes to fill the pool.

She then told her brother that the shape of the path of the water falling is like that of a parabola and also that a parabola can be represented by a quadratic polynomial which has at most two zeroes.
Based on the given information, answer the following questions.
(i) If the product of the zeroes of the quadratic polynomial $f(x)=a\,x^2 – 6x – 6$ is 4, then the value of a.
(ii) The flow of the water in the pool is represented by $x^2 - 2x - 8$, then find its zeroes.
(iii) If α and β be the zeroes of the polynomial $x^2 - 1$, then find the value of $1/\alpha + 1/$.
(iv) Write a quadratic polynomial whose one zero is −3 and product of zeroes is 0.

OTHER QUESTIONS

Q20. Find k if one zero of the quadratic polynomial $2x^2 – 3(k – 4)x – 8$ is negative of the other.

Q21. If one zero of a polynomial $p(x) = x^2 – 4x + 1$ is $2 + \sqrt{3}$, Find the other zero.

Q22. A train covers a distance of 150 km at a uniform speed. If the speed of the train is increased by 5 km/h, it takes 1 hour less for the journey. Find the original speed of the train.
Q23. If the roots of the equation $(a-b)\,x^2 + (b-c)\,x + (c-a) = 0$ are equal, prove that $2a = b + c$
Q24. If α and β are the zeroes of the polynomial $x^2 - 4x + 3$. Then form a quadratic polynomial whose zeroes are 3α and 3β.
Q25. If α and β are the roots of an equation $x^2 + kx + 12 = 0$, then for what value of *k*, if the zeros differ by 1.

Q26. $3x^2$ - 5x + 2 is a quadratic equation with zeros α and β .

Then find a) $\alpha^2 + \beta^2 + 4\alpha\beta$

b) $\alpha^{-1} + \beta^{-1}$

ANSWER KEY

Q1. A)

Q2. A)

Q3. B)

Q4. A)

Q5. A)

Q6. D)

Q7. B)

Q8. C)

Q9. D)

Q10. A)

Q11. D)

Q12. B)

Q13. D)

Q14. A)

Q15. A)

Q16. a) 2 b) 3, 1 c) 18, 0

Q17. i) Given that $h(t) = -16t^2 + 8t + k$. At t = 0, Height is 48 feet.

Putting t = 0, h(t) = 48 in equation $h(t) = -16t^2 + 8t + k$

$48 = -16(0)^2 + 8(0) + k$

48 = k So, the correct answer is (c)

ii) Now, $h(t) = -16t^2 + 8t + 48$ We need to find At what time will she touch the water in the pool . When she touch the water height =0

Putting h = 0 in equation $h(t) = -16t^2 + 8t + 48$

$0 = -16t^2 + 8t + 48$

Dividing by 4 both sides $4t^2 - 2t - 12 = 0$

(4t + 6) (t − 2) = 0

∴ t = (−6)/4=(−**3**)/**2** and t = 2

Since time cannot be negative ∴ t = 2 seconds So, the correct answer is (b)

iii) The required polynomial is $(t - (-1))(t - 2) = t^2 - t - 2$ But, we can multiply any constant to this polynomial, And that polynomial would have the same zeroes

Multiplying by −24

$p(t) = -24 \times (t^2 - t - 2)$

$p(t) = -24t^2 + 24t + 48$ So, the correct answer is (d)

Q18. (i) 1, 3
(ii) k(x2 – 4x + 3) where k is a constant. For example, $p(x) = x^2 – 4x + 3$
(iii) When x = 4, p(4) = 16 – 16 + 3
= 3
(iv) α=7 then, β = (-35)/7 = (-5)
$p(x) = k(x^2 – 2x – 35)$ where k is a constant. For example, $p(x) = x^2 – 2x – 35$

Q19. i) We have, $f(x)=ax^2 - 6x - 6$
Now, Product of zeroes = Constant term/Coefficient of x2
⇒ 4 = (− 6/a)
⇒ a = (−3/2) (ii) We have,

$f(x) = x^2 - 2x – 8$
$= x^2 - 4x + 2x - 8$
$= x(x - 4) + 2(x - 4)$
$= (x - 4)(x + 2)$

f(x) = 0
∴ x = 4,−2
(iii) $\alpha + \beta = 0$, $\alpha\beta = -1$
$1/\alpha + 1/\beta = (\alpha + \beta)/\alpha\beta$
= 0/ (-1)
= 0
(iv) $\alpha = -3$ then,
$\beta = 0/(-3) = 0$
$p(x) = k(x^2 + 3x)$ where k is a constant.

Q20. $\alpha + -\alpha = \frac{3(k-4)}{2}$

$0 = \frac{3(k-4)}{2}$

$k - 4 = 0$

$k = 4$

Q21. Let α and β be the zeroes

α + β = , Let α = 2 + √3

2 + √3 + β = 4

β = 2 - √3

Q22. Let x km/hr be the speed of the train

Distance = 300 km

Given 300/x – 300/(x+5) =2

$2x^2+10x -1500 = 0$

$X^2+5x -750 = 0$

X = -30 or 25

As speed cannot be negative, Speed of the train = 25 km/hr

Q23. Equal roots when $b^2-4ac = 0$

$(b -c)^2 – 4 (a -b) (c -a) = 0$

$b^2 -2bc+c^2 -4ac+4a^2+4bc -4ab = 0$

$4a^2+b^2+c^2-4ac+2bc-4ab = 0$

$(-2a+b+c)^2=0$

$-2a+b+c = 0$

$b+c = 2a$

Q24. x^2 - 21x + 108.

Q25. K =7, -7

Q26. a) 37/9 b) 5/2